Nonlinear dynamics of toroidal Alfven eigenmodes driven by energetic particles

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A kinetic simulation code based on a reduced model is developed to study dynamic evolutions of a single toroidicity-induced shear Alfven eigenmode driven by energetic particles. For zero background damping, it is found that the wave amplitude in nonlinear phase can either saturate for weak energetic particle drives or slowly increase for strong drives. This slow nonlinear growth in strong drive cases is found to be associated with broadening and overlapping of resonances between the wave and trapped particles. For the near-marginal-stability case with a large background damping, the mode nonlinear evolution exhibits strong upward and downward frequency chirping in multiple branches. A hole/clump formation is observed clearly in the corresponding evolution of energetic particle distribution. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4816950]

I. INTRODUCTION

In a nonuniform magnetized plasma, the shear Alfven wave has a continuous spectrum. The continuum can be broken up by toroidicity in a toroidal plasma and a new discrete toroidicity-induced shear Alfven eigenmodes (TAEs) can exist with its frequency inside the toroidicity-induced continuum gaps.1,2 It has been shown that energetic particles can destabilize shear Alfven waves through wave-particle resonant interaction in tokamaks and stellarators. In a tokamak reactor, TAEs can be destabilized by energetic alpha particles.4 Berk et al.5 used a mapping method to investigate the dynamics of energetic particle-driven TAEs and discovered a transition from single mode saturation to multi-mode global quasi-linear diffusion. Pinches et al.6 and Candy et al.7 used a guiding-center Hamiltonian formulation to describe the wave-particle interaction. In their reduced model, the TAE mode structure was held fixed while the amplitude and phase evolve slowly in the time scale much larger than the plasma frequency and background damping. Conclusions are given in Sec. IV.

This article is organized as follows: Sec. II describes the reduced model and the coordinate system used in this work; Sec. III presents the simulation results for cases with/without background damping. Conclusions are given in Sec. IV.

II. SIMULATION MODEL

Following previous work, we used a reduced model in which the mode structure is held fixed in time while the amplitude and the phase evolve slowly in the time scale much longer than TAE oscillation period. An Energetic particle and toroidal Alfven wave interaction Code (EAC) has been developed using this reduced model. The code has been massively parallelized on a variety of computing platforms.

A. Wave evolution

The time advance of the mode amplitude and phase of a TAE is similar to that in Ref. 8. Without loss of generality, the plasma displacement $\xi$ can be written using a two-time-scale formulation as follows:

$$\xi = A(t)\eta(\psi, \theta, n\phi + ot - \omega t),$$

where $\eta$ is a fast-time scale component and satisfies the linearized eigenvalue equation $\omega^2 \rho_B \eta = -\nabla \rho_B + \delta J \times B + J \times \delta B$. The amplitude $A(t)$ and phase $\omega t$ are assumed to vary slowly as compared to the fast mode oscillation. In order to avoid numerical singularity in the phase calculation, two new variables $X = A_0 \cos x$ and $Y = A_0 \sin x$, are introduced. From the momentum equation,23

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Dynamic evolutions of multiple toroidal Alfvén eigenmodes with energetic particles

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In toroidal plasmas, Alfvén waves can be excited by high-energy particles via wave particle resonant interaction in tokamaks. In burning plasmas, the toroidicity-induced shear Alfvén eigenmode (TAE) can be destabilized by energetic alpha particles. Many analytical and numerical studies based on interaction between energetic particles and single Alfvén wave or multiple Alfvén waves have been carried out. In the previous works, frequency chirping of single TAE mode driven by energetic particles with finite background damping has been simulated both in the reduced models and hybrid magnetohydrodynamic simulation. Candy et al. found that the domino effect, which induces enhancement of the energy transfer for interacting mode (multiple mode) case, is not realized because the resonant surfaces are not well separated. It was pointed by Chen et al. that the mode saturation level can be enhanced due to resonance overlapping for well passing particles (μ = 0). The interaction of energetic particles with two different Alfvén eigenmodes (such as TAE/RSAE) has also been investigated recently with the HAGIS code. It was found that the growth rates as well as mode amplitudes can be enhanced significantly by the double-resonant effect.

In our previous single mode simulation, trapped particles were found to be responsible for the slow persistent growth in nonlinear phase for cases with strong energetic particle drive and zero background damping. For the cases near marginal stability with finite background damping, it was found that the frequency chirping exhibits up-down symmetric accompanying with an appearance of hole-clump pair in the energetic particle distribution. In this work, we extend the previous work of single mode to multiple TAE modes. In particular, we investigate the effects of multi-mode interaction on the dynamics of linear and nonlinear evolution. For the cases without background damping, our results show that trapped particles can cause strong mode growth in the nonlinear phase. For the cases near marginal stability with finite background damping, it is found that the multi-mode interaction can significantly affect the nonlinear frequency chirping due to overlapping of holes and clumps in energetic particle distribution.

This article is organized as follows: Sec. II gives the brief description of the simulation code; Sec. III presents the results from multiple mode simulations with/without finite damping. Conclusions are given in Sec. IV.

II. SIMULATION MODEL

We use EAC (Energetic particle and toroidal Alfvén wave interaction Code) code to investigate interaction between energetic particles and multiple Alfvén waves. The wave evolution can be expressed as

\[
X_S = \frac{\left< J_h \cdot \delta E_{x,S} \right>}{2 \left< \frac{\delta^2 E_{x,S}}{\delta x^2} \right> / \rho_0 V^2} - \gamma_X X_S, \quad (1)
\]

\[
Y_S = \frac{\left< J_h \cdot \delta E_{y,S} \right>}{2 \left< \frac{\delta^2 E_{y,S}}{\delta y^2} \right> / \rho_0 V^2} - \gamma_Y Y_S, \quad (2)
\]

where subscript S denotes different modes and \( \delta E = X \delta E_{x,S} + Y \delta E_{y,S} \). Here, two new variables \( X = A \cos \alpha \) and
Nonlinear frequency chirping of toroidal Alfvén eigenmodes in tokamak plasmas

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Abstract

Nonlinear frequency chirping of toroidal Alfvén eigenmodes (TAE) driven by energetic particles is investigated by kinetic simulations in toroidal plasmas. It is found that the up–down symmetry of the frequency chirping of a TAE is broken due to an anisotropic pitch-angle distribution with dominant co-passing energetic particles. The nonuniform distribution of the free energy associated with the initial energetic particle distribution causes biased driving forces that result in a strongly asymmetric frequency chirping. The evolution of the perturbed distribution function in the phase space shows that a hole–clump pair moves together towards the magnetic axis for the small pitch-angle parameter cases. The downward chirping of the mode frequency is associated with the negative drift of the phase island in the KAM surfaces or the resonance $\delta f$ structures in the radial direction. On the other hand, the energetic particle distribution with larger pitch-angle parameters leads to upward chirping of the TAE frequency. The upward chirping is due to the drifting of the resonance structure towards the boundary of the simulation region and overlapping of different poloidal resonances in the $(\Lambda, E)$ phase space at the late stage. The phase space dynamics provides a key mechanism for understanding the wave chirping direction and particle transport process.

Keywords: frequency chirping, TAE, energetic particles, KAM surfaces, wave–particle interaction, hole–clump

(Some figures may appear in colour only in the online journal)

1. Introduction

The Alfvén wave is one of the most fundamental waves in plasmas. In a nonuniform magnetized plasma, the Alfvén wave has a continuous spectrum. The shear Alfvén continuum spectrum in toroidal magnetized plasmas can be broken up by the toroidicity and a discrete toroidicity-induced shear Alfvén eigenmodes (TAE) is generated with frequencies inside the continuum gaps \cite{1,2}. In burning plasmas, energetic particles generated by nuclear fusion or injected by auxiliary heating methods can destabilize TAE \cite{3,4} which can degrade energetic particle confinement and damage the fusion reactor's first wall. With the increase of mode amplitudes driven by wave–particle interaction, the phenomena of TAE frequency chirping are often observed in experiments \cite{5–9}. It should also be noted that the results of this work, although for laboratory fusion plasmas, are also of general interest for understanding wave–particle interactions and nonlinear processes in many other complex systems, such as the energetic electron-driven whistler chorus observed in the Earth’s magnetosphere \cite{10}.

From magnetically confined plasma experiments, it is suggested that there are three different types of frequency chirping of the TAE: up–down chirping; mainly downward chirping; and mainly upward chirping \cite{9}. The up–down symmetric frequency chirping was nicely explained by an analytical model based on a one-dimensional (1D) bump-on-tail problem with finite collision. It was suggested that the up–down symmetric frequency chirping is a result of spontaneous hole–clump formation and drifting in the energetic particle distribution under a marginally unstable regime \cite{11}. The up–down symmetric TAE frequency bifurcation was also reproduced in the reduced kinetic simulations for the marginally unstable regime \cite{12} and a kinetic–MHD hybrid simulation with sources and sinks \cite{13}. Although some numerical simulation results based on the Berk–Breizman (BB) model show the directional frequency chirping and provide a plausible explanation \cite{14,15}, the physical mechanism for the nonlinear frequency chirping direction is still subject to further study in order to understand why only downward or upward chirping is observed in some experiments.

In previous theoretical and numerical studies, isotropic distributions or anisotropic distributions with $\Lambda = 0$ were used to simulate fast frequency chirping for simplicity, where $\Lambda = \mu B_0/E$ is the pitch-angle parameter with $B_0$ the magnetic field strength at the magnetic axis, $\mu$ the magnetic moment and $E$ particle energy. However, for proper modelling
Hybrid simulations of Alfvén modes driven by energetic particles

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A hybrid kinetic-magnetohydrodynamic code (CLT-K) is developed to study nonlinear dynamics of Alfvén modes driven by energetic particles (EP). A \( n = 2 \) toroidicity-induced discrete shear Alfvén eigenmode (TAE)-type energetic particle mode (EPM) with two dominant poloidal harmonics \( (m = 2 \text{ and } 3) \) is first excited and its frequency remains unchanged in the early phase. Later, a new branch of the \( n = 2 \) frequency with a single dominant poloidal mode \( (m = 3) \) splits from the original TAE-type EPM. The new single \( m \) EPM \((m = 3)\) slowly moves radially outward with the downward chirping of the frequency and the mode amplitude remains at a higher level. The original EPM remains at its original position without the frequency chirping, but its amplitude decays with time. Finally, the \( m = 3 \) EPM becomes dominant and the frequency falls into the \( \beta \)-induced gap of the Alfvén continuum. The redistribution of the \( \delta \) in the phase space is consistent with the mode frequency downward chirping and the drifting direction of the resonance region is mainly due to the biased free energy profile. The transition from a TAE-type EPM to a single \( m \) EPM is mainly caused by extension of the \( p = 0 \) trapped particle resonance in the phase space. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4971806]

I. INTRODUCTION

In burning plasmas, the interaction between Alfvén waves and energetic particles which come from nuclear fusion products or auxiliary heating methods is an important issue in current and future magnetic confinement fusion research.\(^1\) Toroidicity-induced discrete shear Alfvén eigenmode (TAE)\(^2,3\) is one of the important Alfvén waves with the frequency located inside the continuum gaps in toroidal plasmas. TAEs can be driven by energetic particles through wave-particle interaction.\(^4,5\) Energetic particle transport and loss associated with TAEs have attracted a lot of attention recently. On the other hand, when wave-particle interactions are strong enough to overcome continuum damping, another non-perturbative mode can be driven in the Alfvén continuum, which is called as Energetic Particle Modes\(^6\) (EPMs). One important example of an EPM is a fishbone instability,\(^7,8\) which can be observed usually in present tokamak experiments and its mechanism is well understood by simulations.\(^9\)

In magnetic confinement plasmas, there are two types of nonlinear scenarios for Alfvén instabilities: a slow frequency sweeping of the mode due to locking to slowly changed plasma equilibrium and a fast frequency chirping associated with a change of the fast ion distribution.\(^10\) It is known that a discrete spectrum of toroidal Alfvén eigenmodes is determined by bulk plasma during the nonlinear stage. The nonlinear behavior of TAEs driven by energetic particles always exhibits a fast frequency chirping in current tokamak experiments.\(^11-14\) Several nonlinear models are developed to understand TAE frequency chirping. Berk and Breizman established a model based on a bump-on-tail problem including a collision term to interpret the frequency chirping resulting from a spontaneous formation of holes and clumps in the energetic particle distribution function.\(^15,16\) Ge Wang and Berk developed a simulation model based on the linear tip model to investigate the frequency chirping of TAEs.\(^17\) It is found that TAEs exist with the frequency both in the gap and continuous spectrum and the TAE frequency chirps towards the upper and lower continua. Using the HAGIS code with a fixed TAE mode structure,\(^18\) Pinches et al. reproduced TAE frequency bifurcations, which agrees well with theoretical prediction and experimental observations.\(^19\) Similar work using an EAC code based on a reduced model is also carried out to investigate hole-clump dynamics in phase space associated with the TAE frequency chirping.\(^20,21\) Many studies to understand physics of the TAE frequency chirping are almost based on semi-analytic 1D models or a reduced model with a fixed mode structure; it lacks a self-consistent simulation to capture the complex properties of the TAE frequency chirping in realistic tokamak plasmas.

It is well known that a magnetohydrodynamic (MHD) model has its limitation in mimicking plasma phenomena induced by energetic particles in which the kinetic effect is very important. In order to solve these problems, a hybrid model is formulated\(^22,23\) to include the wave-particle interaction effect. A lot of numerical simulation codes based on this hybrid model have been developed to investigate energetic particle physics, such as MEGA,\(^24,25\) HMGC,\(^26\) M3D-K,\(^9\) NIMROD,\(^27,28\) and HYM.\(^29,30\) Hybrid codes are powerful tools to investigate different problems of energetic particle effects on MHD activities, such as TAEs,\(^31-37\) fishbone instabilities,\(^9\) EPMs,\(^31,38\) and tearing modes.\(^39\)

In this paper, we develop a new simulation code (CLT-K) which is based on the hybrid kinetic-MHD model to study dynamical evolution of TAEs/EPMs driven by energetic particles. The framework of the MHD part is based on the CLT
Darwin model in plasma physics revisited

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Abstract
Dispersion relations from the Darwin (a.k.a., magnetoinductive or magnetostatic) model are given and compared with those of the full electromagnetic model. Analytical and numerical solutions show that the errors from the Darwin approximation can be large even if phase velocity for a low-frequency wave is close to or larger than the speed of light. Besides missing two wave branches associated mainly with the electron dynamics, the coupling branch of the electrons and ions in the Darwin model is modified to become a new artificial branch that incorrectly represents the coupling dynamics of the electrons and ions.

Keywords: Darwin model, dispersion relation, plasma waves

(Some figures may appear in colour only in the online journal)

1. Introduction
The Darwin (a.k.a., magnetoinductive or magnetostatic) model, which originated from a second order Lagrangian in terms of $v/c$ ($v$ is the velocity of a charged particle and $c$ is the speed of light) for charged particle motion [2], is widely used for low-frequency (nonradiative limit) plasma simulations. Application of this model to plasma simulations was theoretically discussed by Kaufman and Rostler [6]. The widely used particle-in-cell (PIC) simulation in the framework of this model for both 1D and 2D was developed in [1, 9] and can be found in the review papers [3, 5] or textbook [13]. New analytical investigations and applications of the Darwin model can be found in, e.g., [7, 8]. Vlasov-Darwin codes have also been proposed more recently (see e.g., [10]).

It is known that the high-frequency electromagnetic radiations are neglected in Darwin approximation. This is also why Darwin model is useful for simulating low-frequency phenomena, because the larger time step can be used due to neglect of the high-frequency electromagnetic waves. However, how the low-frequency wave is affected by this assumption is not quite clear. Most existing papers focused on the simulation aspects. No one made a careful study of the quantitative differences of the results between the Darwin approximation and the full electromagnetic (EM) model. Busnardo-Neto et al [1] only benchmarked a simplified dispersion relation [4] under the cold plasma assumption with the Darwin model. The properties of the dispersion relations from the Darwin model had been discussed in [6, 13].

In this work, we give the limits of validity of the Darwin model by carefully comparing the dispersion relations from the Darwin model and the full EM model.

2. Basic equations
All field variables can be divided into two parts in the Darwin model [1]: the transverse ($T$, divergence free) part and the longitudinal ($L$, curl free) part.

$$E = E_T + E_L, \quad V \cdot E_T = 0, \quad V \times E_L = 0,$$
$$B = B_T, \quad V \cdot B_T = 0,$$
$$J = J_T + J_L, \quad V \cdot J_T = 0, \quad V \times J_L = 0. \quad (1)$$

The Maxwell equations are

$$\nabla \cdot E_T = \frac{\rho}{\varepsilon_0},$$
$$\nabla \cdot B = 0,$$
$$V \times E_T = - \frac{\partial B}{\partial t} \text{ (or, } V \times E = - \frac{\partial B}{\partial t}),$$
$$V \times B = \mu_0 J + \frac{\mu_0}{\varepsilon_0} \partial E_L/\partial t + \frac{\mu_0}{\varepsilon_0} \partial E_T/\partial t \quad \text{dropped in Darwin model.} \quad (2)$$

For the PIC simulation, the governing equations are as
Destabilization of reversed shear Alfvén eigenmodes driven by energetic ions during NBI in HL-2A plasmas with $q_{\text{min}} \sim 1$


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Abstract

Two groups of frequency sweeping modes are observed and interpreted in the HL-2A plasmas with $q_{\text{min}} \sim 1$. The tokamak simulation code calculations indicate the presence of a reversed shear $q$-profile during the existence of these modes. The mode frequencies lie in between TAE and BAE frequencies, i.e. $\omega_{\text{BAE}} < \omega < \omega_{\text{TAE}}$, and these modes are highly localized near $q_{\text{min}}$, i.e. $r/a \sim 0.25$. A group of modes characterized by down-sweeping frequency with $q_{\text{min}}$ decrease due to $q_{\text{min}} > 1$ and $nq_{\text{min}} - m > 0$, and another group of modes characterized by up-sweeping frequency with $q_{\text{min}}$ drop, owing to $q_{\text{min}} < 1$ and $nq_{\text{min}} - m < 0$ before sawtooth crash. The kinetic Alfvén eigenmode code analysis supports that the down-sweeping modes are kinetic reverse shear Alfvén eigenmodes (KRSAEs), and the up-sweeping modes are RSAEs, which exist in the ideal or kinetic MHD limit. In addition, the down- and up-sweeping RSAEs both have fast nonlinear frequency behaviour in the process of slow frequency sweeping, i.e. producing pitch-fork phenomena. These studies provide valuable constraint conditions for the $q$-profile measurements.

Keywords: frequency sweeping modes, RSAE, Alfvén continuum, energetic ions

(Some figures may appear in colour only in the online journal)

1. Introduction

Alfvén modes are MHD instabilities that are driven unstable by the energetic particles in a fusion reactor, such as ITER and DEMO, and they may induce excessive alpha particle losses [1–2]. Understanding and control of these instabilities will be necessary for the successful operation of such experiments. A well-known discrete Alfvén mode is the reverse shear Alfvén eigenmode (RSAE) or Alfvén cascade (AC) [3–4]. The RSAEs imply the presence of a reversed shear $q$-profile, and as a result the RSAE phenomenon can serve as a useful plasma diagnostic for the measurement of safety-factor profiles, i.e. so-called MHD spectroscopy [5]. Therefore it is important to study the properties of RSAEs, such as the existence and stability of the modes, as well as energetic-particle losses induced by the modes. In the toroidal plasmas with reversed shear profiles, the RSAE can exist with frequencies just above the maximum of Alfvén continuum. Kinetic effects such as ion finite Larmor radius and perturbed parallel electric field can lead to kinetic RASEs (KRSAEs) [6–7]. To our knowledge, as the value of $q_{\text{min}}(t)$ changes during the discharge, the RSAE frequency changes at a rate [8],

$$\frac{d}{dt}\omega_{\text{RSAE}}(t) \approx \pm m \frac{v_A}{R} \frac{d}{dt}q_{\text{min}}(t).$$

Equation (1) represents an important distinctive feature of RSAEs, and the plus and minus signs correspond to the downward and upward frequencies of RSAEs respectively. The RSAE frequency can be expressed by the formula,

$$\omega_{\text{RSAE}}^2 = \frac{V_A^2}{R^2} \left( n - \frac{m}{q_{\text{min}}} \right)^2 + \omega_{\text{BAE}}^2 + \delta \omega^2.$$