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L^2 limits of generalized Jiřina processes

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ABSTRACT

A kind of L^2-convergence theorems for the state dependent branching processes with continuous state and discrete time is studied and a group of sufficient conditions is obtained.

1. Introduction

In this paper, the term "generalized Jiřina process" is used to refer to a time-homogeneous Markov process $X = \{X_n\}$ with the state space $[0, +\infty)$ such that for every $\lambda \geq 0$ and $x \geq 0$,

$$\mathbb{E}(e^{-\lambda X_{n+1}} | X_n = x) = \exp\{-xF(x, \lambda)\},$$

where the reproduction cumulative function (r.c.f)

$$F(x, \lambda) = \gamma(x)\lambda + \int_0^{+\infty} (1 - e^{-\lambda u})\nu(x, du),$$

$\gamma(x)$ is a non-negative Borel function and $(1 \wedge u)\nu(x, du)$ is a finite kernel from $[0, +\infty)$ to $(0, +\infty)$. The generalized Jiřina process is determined by the function $F(x, \lambda)$ or the pair of functions $\gamma(x)$ and $\nu(x, du)$ which, for convenience, are called the drift and the Lévy measure, respectively. By convention, we refer the "offspring mean" to the averaged conditional moment function

$$m(x) = \mathbb{E}(X_1 | X_0 = x)/x = \gamma(x) + \int_0^{+\infty} u\nu(x, du) \quad \text{for } x > 0, \tag{1.1}$$

and say the generalized Jiřina process to be subcritical, critical or supercritical, if $\lim_{x\to+\infty} m(x) < 1, = 1$ or > 1, respectively.

The generalized Jiřina process was firstly introduced by the second author (Li, 2006) in 2006, where it was proved that this kind of process can naturally arise as the functional limit of a kind of state-dependent Galton–Watson processes, more precisely, the population-size-dependent branching process (see Klebaner, 1984b). This kind of processes can approach the

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Brownian motion between two random trajectories

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Abstract: Consider the first exit time of one-dimensional Brownian motion $\{B_s\}_{s\geq 0}$ from a random passageway. We discuss a Brownian motion with two time-dependent random boundaries in quenched sense. Let $\{W_s\}_{s\geq 0}$ be an other one-dimensional Brownian motion independent of $\{B_s\}_{s\geq 0}$ and let $\mathbb{P}(\cdot|W)$ represent the conditional probability depending on the realization of $\{W_s\}_{s\geq 0}$. We show that

$$-t^{-1}\ln \mathbb{P}^x(\forall_{s\in[0,t]} a+\beta W_s \leq B_s \leq b+\beta W_s|W)$$

converges to a finite positive constant $\gamma(\beta)(b-a)^{-2}$ almost surely and in L^p $(p\geq 1)$ if $a < B_0 = x < b$ and $W_0 = 0$. When $\beta = 1, a+b = 2x$, it is equivalent to the random small ball probability problem in the sense of equiditribution, which has been investigated in [4]. We also find some properties of the function $\gamma(\beta)$. An important moment estimation has also been obtained, which can be applied to discuss the small deviation of random walk with random environment in time (see [12]).

Keywords: Brownian motion, First exit time, Random boundary, Limit theorem.
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1. Introduction

The first exit time of Brownian motion is a classic and interesting topic which has been researched by many scholars. Let us first recall a very basic result in this field. For a standard Brownian motion $\{B_t\}_{t\geq 0}$ starting from x, it is known that

$$\lim_{t\to+\infty}\frac{-\ln \mathbb{P}^x(\forall_{s\leq t} a\leq B_s\leq b)}{t}=\frac{\pi^2}{2(b-a)^2}, \tag{1.1}$$

where $a < x < b$. (1.1) shows that the first exit time from a bounded interval has negative exponential tail distribution and the coefficient depends on the width of the bounded interval.

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