Note

The maximum girth and minimum circumference of graphs with prescribed radius and diameter

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ABSTRACT

Ostrand posed the following two questions in 1973. (1) What is the maximum girth of a graph with radius \( r \) and diameter \( d \)? (2) What is the minimum circumference of a graph with radius \( r \) and diameter \( d \)? Question 2 has been answered by Hrnčiar who proves that if \( d \leq 2r - 2 \) the minimum circumference is \( 4r - 2d \). In this note we first answer Question 1 by proving that the maximum girth is \( 2r + 1 \). This improves on the obvious upper bound \( 2d + 1 \) and implies that every Moore graph is self-centered. We then prove a property of the blocks of a graph which implies Hrnčiar’s result.

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1. Introduction

We consider finite simple graphs. Ostrand [6, p.75] posed the following two questions in 1973.

**Question 1.** What is the maximum girth of a graph with radius \( r \) and diameter \( d \)?

**Question 2.** What is the minimum circumference of a graph with radius \( r \) and diameter \( d \)?

**Question 2** has been answered by Hrnčiar [5] who proves that if \( d \leq 2r - 2 \) the minimum circumference is \( 4r - 2d \). In this note we first answer Question 1 by proving that the maximum girth is \( 2r + 1 \). This improves on the obvious upper bound \( 2d + 1 \) and implies that every Moore graph is self-centered. We then prove a property of the blocks of a graph which implies Hrnčiar’s result.

Google shows 63 citations of Ostrand’s paper [6] and MathSciNet shows 7 citations. It seems that Question 1 has not been treated.

For terminology and notations we follow the books [1,3,8]. We denote by \( V(G) \) the vertex set of a graph \( G \) and by \( d(u, v) \) the distance between two vertices \( u \) and \( v \). The eccentricity, denoted by \( e(v) \), of a vertex \( v \) in a graph \( G \) is the distance to a vertex farthest from \( v \). Thus \( e(v) = \max\{d(v, u) \mid u \in V(G)\} \). If \( e(v) = d(v, x) \), then the vertex \( x \) is called an eccentric vertex of \( v \). The radius of a graph \( G \), denoted \( \text{rad}(G) \), is the minimum eccentricity of all the vertices in \( V(G) \), whereas the diameter of \( G \), denoted \( \text{diam}(G) \), is the maximum eccentricity. A vertex \( v \) is a central vertex of \( G \) if \( e(v) = \text{rad}(G) \). When \( H \) is a subgraph of a graph \( G \) and \( u, v \in V(H) \), \( d_H(u, v) \) and \( e_H(v) \) will mean the distance and eccentricity in \( H \) respectively.

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On Vertex Types of Graphs

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Abstract
The vertices of a graph are classified into seven types by J.T. Hedetniemi, S.M. Hedetniemi, S.T. Hedetniemi and T.M. Lewis and they ask the following questions: (1) What is the smallest order \( n \) of a graph having \( n - 2 \) very typical vertices or \( n - 2 \) typical vertices? (2) What is the smallest order of a pantypical graph? We answer these two questions and determine all the possible orders of the graphs in these three classes in this paper.

Keywords Graph · Vertex type · Degree · Smallest order

1 Introduction
We consider finite simple graphs. For a vertex \( v \) in a graph, we denote by \( d(v) \) and \( N(v) \) the degree of \( v \) and the neighborhood of \( v \) respectively throughout the paper. Motivated by the notions of strong and weak vertices in [3] and [2], Hedetniemi, Hedetniemi, Hedetniemi and Lewis [1] classified the vertices of a graph into the following seven types.

Definition A vertex \( u \) in a simple graph is said to be

1. very strong if \( d(u) \geq 2 \) and for every vertex \( v \in N(u) \), \( d(u) > d(v) \);
2. strong if \( d(u) \geq 2 \) and for every vertex \( v \in N(u) \), \( d(u) \geq d(v) \), at least one neighbor \( x \in N(u) \) has \( d(x) < d(u) \) and at least one neighbor \( y \in N(u) \) has \( d(y) = d(u) \);
3. regular if \( d(u) \geq 0 \) and for every vertex \( v \in N(u) \), \( d(u) = d(v) \);

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Algebraically positive matrices

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\textbf{A B S T R A C T}

We introduce the concept of algebraically positive matrices and investigate some basic properties, including a characterization, the index of algebraic positivity, and sign patterns that allow or require this property. We also pose two open problems.

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1. Introduction

A positive (nonnegative) matrix is a matrix all of whose entries are positive (nonnegative) real numbers. The notation $A > 0$ means that $A$ is a positive matrix. We introduce the following concept and study its basic properties.

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