Prokhorov distance with rates of convergence under sublinear expectations[1)]

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Abstract Prokhorov distances under sublinear expectations are presented in CLT and functional CLT, and the convergence rates for them are obtained by Lindeberg method. In particular, the obtained estimate in functional CLT yields known Borovkov's estimate in classical functional CLT with explicit constant.

Keywords Sublinear expectation; Prokhorov distance; Lindeberg method

Mathematics Subject Classification (2010) Primary: 60F05

1 Introduction

1.1. Convergence of a sequence of random variables is an important concept in probability theory and its applications. Convergence in distribution is the weakest form of such convergences. However, convergence in distribution is very frequently used in practice, one of the most important application is the central limit theorem (CLT). It is well known that if we have classical i.i.d. random variables $X_1, X_2, \ldots$ with

$$\mathbf{E}[X_1] = \mu, \quad 0 < \sigma^2 = \mathbf{Var}X_1 = \mathbf{E}[(X_1 - \mu)^2] < \infty, \tag{1}$$

then (all limits in the paper are taken as $n \longrightarrow \infty$)

$$W_n := \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i - \mu}{\sigma} \Rightarrow Z \sim N(0,1), \tag{2}$$

where $Z \sim N(0,1)$ means that Z has the standard normal distribution. In particular, convergence (2) is equivalent to the fact that for all bounded and

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Convergences of Random Variables Under Sublinear Expectations*

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Abstract In this note, the authors survey the existing convergence results for random variables under sublinear expectations, and prove some new results. Concretely, under the assumption that the sublinear expectation has the monotone continuity property, the authors prove that convergence in capacity is stronger than convergence in distribution, and give some equivalent characterizations of convergence in distribution. In addition, they give a dominated convergence theorem under sublinear expectations, which may have its own interest.

Keywords Sublinear expectation, Capacity, The dominated convergence theorem
2000 MR Subject Classification 60J45, 60G51

1 Introduction

It is well known that limit theory plays an important role in probability theory and statistics. Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\{X, X_n, n \geq 1\}$ be a sequence of random variables. Then we have the following convergences:

(1) $\{X_n, n \geq 1\}$ is said to almost surely converge to X, if there exists a set $N \in \mathcal{F}$ such that $P(N) = 0$ and $\forall \omega \in \Omega \backslash N$, $\lim\limits_{n\to\infty} X_n(\omega) = X(\omega)$, which is denoted by $X_n \xrightarrow{\text{a.s.}} X$ or $X_n \to X$ a.s.

(2) $\{X_n, n \geq 1\}$ is said to converge to X in probability, if for any $\varepsilon > 0$, $\lim\limits_{n\to\infty} P(\{|X_n - X| \geq \varepsilon\}) = 0$, which is denoted by $X_n \xrightarrow{P} X$.

(3) $\{X_n, n \geq 1\}$ is said to L^p converge to X $(p > 0)$, if $\lim\limits_{n\to\infty} E[|X_n - X|^p] = 0$, which is denoted by $X_n \xrightarrow{L^p} X$.

(4) $\{X_n, n \geq 1\}$ is said to converge to X in distribution, if for any bounded continuous function f, $\lim\limits_{n\to\infty} E[f(X_n)] = E[f(X)]$, which is denoted by $X_n \xrightarrow{d} X$.

(5) $\{X_n, n \geq 1\}$ is said to completely converge to X, if for any $\varepsilon > 0$, $\sum\limits_{n=1}^{\infty} P(\{|X_n - X| \geq \varepsilon\}) < \infty$, which is denoted by $X_n \xrightarrow{\text{c.c.}} X$ (see [4]).

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A Note on Uniform Integrability of Random Variables in a Probability Space and Sublinear Expectation Space *

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Abstract: In this note we discuss uniform integrability of random variables. In a probability space, we introduce two new notions on uniform integrability of random variables, and prove that they are equivalent to the classic one. In a sublinear expectation space, we give de La Vallée Poussin criterion for the uniform integrability of random variables and do some other discussions.

Keywords: uniform integrability; sublinear expectation

2010 Mathematics Subject Classification: 60F25; 28A25

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§1. Introduction

It is well known that the uniform integrability of a family of random variables plays an important role in probability theory. As to the uniform integrability criterions, please refer to [1; P. 96], [2], [3; P. 94], [4], [5; P. 138] and [6].

In [7], the authors introduced the notion of a sequence of random variables being *uniformly nonintegrable* and gave some interesting characterizations of this uniform nonintegrability. In [8], a weak notion of a sequence of random variables being uniformly nonintegrable was introduced and some equivalent characterizations were given. Motivated from [7] and [8], we will introduce two new notions of a sequence of random variables being uniformly integrable in a probability space, and prove that they are equivalent to the classic one.

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